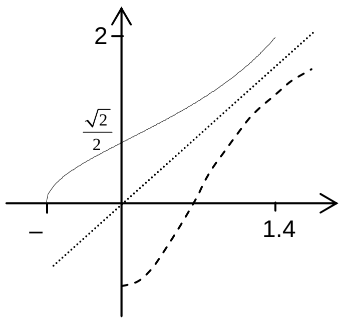
4753 (C3) Methods for Advanced Mathematics

1 $e^{2x} - 5e^x = 0$ $\Rightarrow e^x(e^x - 5) = 0$ $\Rightarrow e^x = 5$ $\Rightarrow x = \ln 5$ or 1.6094	M1 M1 A1 A1 [4]	factoring out e^x or dividing $e^{2x} = 5e^x$ by e^x $e^{2x} / e^x = e^x$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
or $\ln(e^{2x}) = \ln(5e^x)$ $\Rightarrow 2x = \ln 5 + x$ $\Rightarrow x = \ln 5$ or 1.6094	M1 A1 A1 A1 [4]	taking lns on $e^{2x} = 5e^x$ $2x$, $\ln 5 + x$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
2 (i) When $t = 0$, $T = 100$ $\Rightarrow 100 = 20 + b$ $\Rightarrow b = 80$ When $t = 5$, $T = 60$ $\Rightarrow 60 = 20 + 80e^{-5k}$ $\Rightarrow e^{-5k} = ½$ $\Rightarrow k = \ln 2 / 5 = 0.139$	M1 A1 M1 A1 [4]	substituting $t = 0$, $T = 100$ cao substituting $t = 5$, $T = 60$ 1/5 ln 2 or 0.14 or better
(ii) $50 = 20 + 80e^{-kt}$ $\Rightarrow e^{-kt} = 3/8$ $\Rightarrow t = \ln(8/3) / k = 7.075$ mins	M1 A1 [2]	Re-arranging and taking lns correctly – ft their b and k answers in range 7 to 7.1
3(i) $\frac{dy}{dx} = \frac{1}{3}(1+3x^2)^{-2/3}.6x$ $= 2x(1+3x^2)^{-2/3}$	M1 B1 A1 [3]	chain rule $1/3\, u^{-2/3}$ or $\frac{1}{3}(1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer
(ii) $3y^2 \frac{dy}{dx} = 6x$ $\Rightarrow dy/dx = 6x/3y^2$ $= \frac{2x}{(1+3x^2)^{2/3}} = 2x(1+3x^2)^{-2/3}$	M1 A1 A1 E1 [4]	$3y^2 \frac{dy}{dx}$ $= 6x$ if deriving $2x(1+3x^2)^{-2/3}$, needs a step of working

4(i) $\int_0^1 \frac{2x}{x^2+1}\,dx = \left[\ln(x^2+1)\right]_0^1$ $= \ln 2$	M2 A1 [3]	$[\ln(x^2 + 1)]$ cao (must be exact)
or let $u = x^2 + 1$, $du = 2x\,dx$ $\Rightarrow \int_0^1 \frac{2x}{x^2+1}\,dx = \int_1^2 \frac{1}{u}\,du$ $= [\ln u]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u}\,du$ or $\left[\ln(1+x^2)\right]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1}\,dx = \int_0^1 \frac{2x+2-2}{x+1}\,dx = \int_0^1 (2 - \frac{2}{x+1})\,dx$ $= [2x - 2\ln(x+1)]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x + 1)$ 2, $-2/(x+ 1)$
or $\int_0^1 \frac{2x}{x+1}\,dx$ let $u = x + 1$, $\Rightarrow du = dx$ $= \int_1^2 \frac{2(u-1)}{u}\,du$ $= \int_1^2 (2 - \frac{2}{u})\,du$ $= [2u - 2\ln u]_1^2$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$) *and* correct limits used for u or x $2(u - 1)/u$ dividing through by u $2u - 2\ln u$ allow ft on $(u - 1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)
5 (i) $a = 0$, $b = 3$, $c = 2$	B(2,1,0)	or $a = 0$, $b = -3$, $c = -2$
(ii) $a = 1$, $b = -1$, $c = 1$ *or* $a = 1$, $b = 1$, $c = -1$	B(2,1,0) [4]	
6 $f(-x) = -f(x)$, $g(-x) = g(x)$ $g\,f(-x) = g\,[-f\,(x)]$ $= g\,f\,(x)$ $\Rightarrow$ g f is even	B1B1 M1 E1 [4]	condone f and g interchanged forming $gf(-x)$ or $gf(x)$ and using $f(-x) = -f(x)$ www
7 Let $\arcsin x = \theta$ $\Rightarrow x = \sin\theta$ $\theta = \arccos y \Rightarrow y = \cos\theta$ $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow x^2 + y^2 = 1$	 M1 M1 E1 [3]	

8(i) At P, $x \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 0$ $\Rightarrow$ $3x = \pi/2, 3\pi/2$ $\Rightarrow$ $x = \pi/6, \pi/2$ So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	M1 M1 A1 A1 [4]	or verification $3x = \pi/2, (3\pi/2\ldots)$ dep both Ms condone degrees here
(ii) $\frac{dy}{dx} = -3x\sin 3x + \cos 3x$ At P, $\frac{dy}{dx} = -\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = -\frac{\pi}{2}$ At TPs $\frac{dy}{dx} = -3x\sin 3x + \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 3x \sin 3x$ $\Rightarrow$ $1 = 3x \sin 3x / \cos 3x = 3x \tan 3x$ $\Rightarrow$ $x \tan 3x = 1/3$ *	M1 B1 A1 M1 A1cao M1 E1 [7]	Product rule $d/dx (\cos 3x) = -3 \sin 3x$ cao (so for $dy/dx = -3x\sin3x$ allow B1) mark final answer substituting their $-\pi/6$ (must be rads) $-\pi/2$ $dy/dx = 0$ and $\sin 3x / \cos 3x = \tan 3x$ used www
(iii) $A = \int_0^{\pi/6} x\cos 3x\,dx$ Parts with $u = x$, $dv/dx = \cos 3x$ $du/dx = 1$, $v = 1/3 \sin 3x$ $\Rightarrow$ $A = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \int_0^{\pi/6}\frac{1}{3}\sin 3x\,dx$ $= \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{18} - \frac{1}{9}$	B1 M1 A1 A1 M1dep A1 cao [6]	Correct integral and limits (soi) – ft their P, but must be in radians can be without limits dep previous A1. substituting correct limits, dep 1st M1: ft their P provided in radians o.e. but must be exact

9(i) $f'(x) = \frac{(x^2+1)4x - (2x^2-1)2x}{(x^2+1)^2}$ $= \frac{4x^3+4x-4x^3+2x}{(x^2+1)^2} = \frac{6x}{(x^2+1)^2}$ * When $x > 0$, $6x > 0$ and $(x^2 + 1)^2 > 0$ $\Rightarrow$ $f'(x) > 0$	M1 A1 E1 M1 E1 [5]	Quotient or product rule correct expression www attempt to show or solve f'(x) > 0 numerator > 0 and denominator > 0 or, if solving, $6x > 0 \Rightarrow x > 0$
(ii) $f(2) = \frac{8-1}{4+1} = 1\frac{2}{5}$ Range is $-1 \le y \le 1\frac{2}{5}$	B1 B1 [2]	 must be $\le$, y or f(x)
(iii) $f'(x)$ max when $f''(x) = 0$ $\Rightarrow$ $6 - 18x^2 = 0$ $\Rightarrow$ $x^2 = 1/3$, $x = 1/\sqrt{3}$ $\Rightarrow$ $f'(x) = \frac{6/\sqrt{3}}{(1\frac{1}{3})^2} = \frac{6}{\sqrt{3}}.\frac{9}{16} = \frac{9\sqrt{3}}{8} = 1.95$	M1 A1 M1 A1 [4]	 (±)1/√3 oe (0.577 or better) substituting 1/√3 into f'(x) 9√3/8 o.e. or 1.95 or better (1.948557..)
(iv) Domain is $-1 < x < 1\frac{2}{5}$ Range is $0 \le y \le 2$ 	B1 B1 M1 A1 cao [4]	ft their 1.4 but not $x \ge -1$ or $0 \le g(x) \le 2$ (not f) Reasonable reflection in $y = x$ from (–1, 0) to (1.4, 2), through (0, √2/2) allow omission of one of –1, 1.4, 2, √2/2
(v) $y = \frac{2x^2-1}{x^2+1}$ $x \leftrightarrow y$ $x = \frac{2y^2-1}{y^2+1}$ $\Rightarrow$ $xy^2 + x = 2y^2 - 1$ $\Rightarrow$ $x + 1 = 2y^2 - xy^2 = y^2(2 - x)$ $\Rightarrow$ $y^2 = \frac{x+1}{2-x}$ $\Rightarrow$ $y = \sqrt{\frac{x+1}{2-x}}$ *	 M1 M1 M1 E1 [4]	(could start from g) Attempt to invert clearing fractions collecting terms in y^2 and factorising www

NOTE

For the domain of g(*x*) in 9(iv) the answer is -1£x £ 1.4. The published markscheme gives -1 < x < 1.4; this is because, at the time, the published markscheme was not the same as the final one used by the examiners and so the published markscheme contains a printing error. This did not affect marking as the examiners used a correct version.